Observer-based networked predictive controller design with an application to automotive drivetrains

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Abstract. State feedback control is very attractive due to the precise computation of the gain matrix, but the implementation of a state feedback controller is possible only when all state variables are directly measurable. This condition is almost impossible to accomplish due to the excess number of required sensors or unavailability of states for measurement in most of the practical situations. Hence the need for an estimator or observer is obvious to estimate all the state variables. As such, the goal of this paper is to provide a control design methodology based on a Luenberger observer design that can assure the closed-loop performances of a vehicle drivetrain with backlash, while compensating the network-induced time-varying delays. The designed control strategy was experimentally tested on a vehicle drivetrain emulator controlled through Controller Area Network.

Keywords: Design methodologies, networked control systems, observer design, predictive control.

1 Introduction

The advanced control techniques, which rely on full state feedback, rather than classic output feedback became increasingly attractive for different control applications in the last years. However, the implementation of a state feedback controller is possible only when all state variables are directly measurable. This condition is almost impossible to accomplish due to the excess number of required sensors or unavailability of states for measurement in most of the practical situations. Hence the need for an estimator or observer is obvious to estimate the state variables by observing the input and the output of the controlled system. While observer design for linear discrete-time systems is a trivial problem, observer design for piecewise linear plant models raises considerable difficulties [1]. In the literature there are papers that address the design of Luenberger type observers for continuous-time switched affine systems [2], bi-modal piecewise linear systems in both continuous and discrete time [3], discrete-time systems with piecewise affine (PWA) dynamics [4], discrete-time systems with input-induced bilinearity [1].

The problem considered in this paper is the development of a networked predictive control strategy, based on the design of a Luenberger observer, which is applied to minimize the effects of the backlash of a vehicle drivetrain, while compensating the controller are network (CAN) induced time-varying delays. The proposed solution consists of three steps: firstly, a general framework of modeling the network-induced time-varying delays as disturbances for a piecewise linear (PWL) model is presented; secondly, a Luenberger observer which estimates all the state variables is synthesized for the discrete-time system with PWL dynamics; thirdly, a robust one step ahead model predictive control (MPC) scheme is designed using the concept of flexible control Lyapunov functions (CLF) that explicitly accounts for rejection of disturbances introduced by the time-varying delays in the communication network and guarantees also the input-to-state stability of the system in a non-conservative way. The full state-feedback predictive control strategy based on the Luenberger observer design was implemented in Matlab/Simulink to control the physical plant on the real-time simulation test-bench and the designed experiments validate the proposed approach.

2 Network delay modeled as disturbance

Consider the standard network control system (NCS) illustrated in Fig. 1, which is composed of five parts: a communication network, a physical plant, one sensor node (S), one controller node and one actuator node (A). The delays introduced by the communication network, which can be smaller and larger than a sampling period, are represented as τ^{ca} for the delay in the forward channel and as τ^{sc} for the delay in the feedback channel. It is easy to observe that Fig. 1 is equivalent to Fig. 2, in which the difference between the actual control signal and the delayed one and the difference between the actual output and the delayed one are regarded as hypothetical disturbances [5].

Let a_k and b_k denote the delay in the forward channel and in the feedback channel, respectively, at time instant k, expressed as a number of sampling periods: $a_k = \lceil \tau_k^{ca}/T_s \rceil$, $b_k = \lceil \tau_k^{sc}/T_s \rceil$, where T_s is the sampling period of the system. Moreover, let $\bar{a}$ and $\bar{b}$ denote the maximum delays in the forward channel and in the feedback channel, respectively, expressed as a number of sampling periods: $\bar{a} = \lceil \tau^{ca^{max}}/T_s \rceil$, $\bar{b} = \lceil \tau^{sc^{max}}/T_s \rceil$.

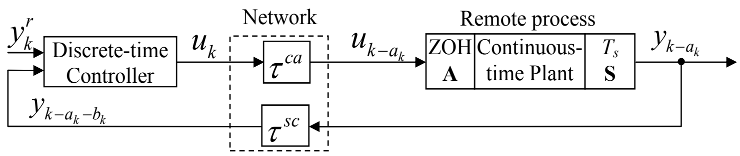

Fig. 1. Control system with network-induced time delay.

The physical plant is given by the discrete-time state-space PWL model [6]

$$\begin{cases} x_{k+1} = A_{di}x_k + B_{di}u_k + \sum_{j=0}^{a_k} \Delta_{j,k}(u_{k-j-1} - u_{k-j}), \text{ if } x_k \in \mathbb{P}_i \\ y_k = C_d x_k \end{cases}, \quad (1)$$

where x_k and u_k are the system state and the control signal, respectively, $A_{di} = e^{A_{ci}T_s}$, $B_{di} = \int_0^{T_s} e^{A_{ci}(T_s-\theta)} d\theta B_{ci}$ and $C_d = C_c$ are the discrete-time system matrices, A_{ci},

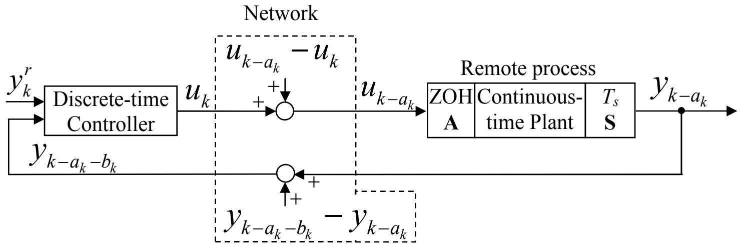

Fig. 2. Control system with network-induced time delay as disturbance.

B_{ci} and C_c are the continuous-time system matrices and

$$\Delta_{j,k} = \begin{cases} 0, & \tau^{ca}_{k-j} - jT_s \leq 0 \\ \int_0^{\tau^{ca}_{k-j} - jT_s} e^{A_{ci}(T_s-\theta)} d\theta B_{ci}, & 0 < \tau^{ca}_{k-j} - jT_s < T_s \\ \int_0^{T_s} e^{A_{ci}(T_s-\theta)} d\theta B_{ci}, & T_s \leq \tau^{ca}_{k-j} - jT_s \end{cases} \quad (2)$$

for all $k \in \mathbb{Z}_+$ and $j \in \mathbb{Z}_{[0,a_k]}$ with

$$\tau^{ca}_k \geq \tau^{ca}_{k-1} - T_s. \quad (3)$$

Please note that $i \in \mathcal{I} := \mathbb{Z}$ denotes the active mode at time $k \in \mathbb{Z}_+$. The collection of sets $\{\mathbb{P}_i | i \in \mathcal{I}\}$ defines a partition of the state-space $\mathbb{X} \subseteq \mathbb{R}^n$.

The maximum discrete-time forward channel disturbance representation is given by

$$u^d_k = u_{k-a_k} - u_k \quad (4)$$

and it is shown in [5] that a bounded set $\mathbb{W}_u$ can be found, in which to include all possible disturbances that appear due to the time-varying delays introduced in the forward channel, knowing that the input of the plant is bounded.

The hypothetical disturbance exerted on the physical plant delays the feedback signal, the discrete-time feedback channel disturbance representation being given by

$$y^d_k = y_{k-a_k-b_k} - y_{k-a_k}. \quad (5)$$

and it is shown in [5] that a bounded set $\mathbb{W}_x$ can be found, in which to include all possible disturbances that can appear due to the time-varying delays introduced in the feedback channel, knowing that the input of the remote process is bounded.

Now, the output that reaches the controller becomes

$$y_k = Cx_k = C(A_{di}x_{k-1} + B_{di}u_{k-1}) + w_k, \quad (6)$$

where $w_k = C(B_{di}u^d_{k-1}) + y^d_k$.

Even though the time-varying delay gives a time-varying disturbance, the sets $\mathbb{W}_u$, which is defined by $C(B_{di}u^d_{k-1})$, and $\mathbb{W}_y$, which is defined by y^d_k, remain fixed, so this modeling technique is suitable for the use of the results presented in [7], in which the disturbances are explicitly taken into account during the design phase of the predictive controller, which will be accomplished in the next section.

3 Predictive controller with Luenberger observer design

3.1 Luenberger observer design

This section provides a Luenberger type observer synthesis technique for the PWL vehicle drivetrain model given in the previous section.

Assumption *The region of the discrete-time PWL model $\mathbb{P}_i$, such that $\begin{bmatrix} \hat{x} \\ u \\ w \end{bmatrix} \in \mathbb{P}_i$, is known at each moment of time.*

The Luenberger observer can be written as follows:

$$\begin{cases} \hat{x}_{k+1} = A_{di}\hat{x}_k + B_{di}u_k + w_k + L_i(y - \hat{y}), \\ \hat{y} = C_d\hat{x}. \end{cases} \tag{7}$$

From the above assumption it follows that for any $\begin{bmatrix} \hat{x} \\ u \\ w \end{bmatrix} \in \mathbb{P}_i$, $\begin{bmatrix} x \\ u \\ w \end{bmatrix} \in \mathbb{P}_i$ and as such, the error dynamics is

$$e_{k+1} = (A_{di} - L_iC_d)e_k. \tag{8}$$

Theorem 1. *Suppose there exists a positive definite matrix P and a number $\rho \in \mathbb{R}_{(0,1)}$ and*

$$(A_{di} - L_iC_d)^\top P(A_{di} - L_iC_d) \preceq \rho P, \tag{9}$$

hold for all $i \in \mathbb{Z}_{[1,s]}$. Then the error dynamics (8) *is globally asymptotically stable.*

To solve the inequality (9) in Theorem 1 the following lemma is introduced.

Lemma 1. *Suppose that*

$$\begin{bmatrix} \rho P & A_{di}^\top P - C_d^\top Y_i \\ PA_{di} - Y_i^\top C_d & P \end{bmatrix} \preceq 0 \tag{10}$$

holds for all $i \in \mathbb{Z}_{[1,s]}$ and some $P \succ 0, \rho \in \mathbb{R}_{(0,1)}$ and Y_i. Then the inequality (9) *holds with P, ρ and $L_i = (Y_iP^{-1})^\top$, respectively.*

For the proof of Theorem 1 and Lemma 1 the interested reader is referred to Theorem 1 and Lemma 1 from [2]. Note that it is possible to compute a single gain $L = L_i$ for the discrete-time PWL observer by considering a single $Y = Y_i$ in (10).

3.2 Robust one step ahead predictive controller design

Consider the perturbed discrete-time constrained nonlinear drivetrain system (6) with the observer (7). Naturally, it is assumed that the set of feasible states $\mathbb{X}$, the set of feasible inputs $\mathbb{U}$ and the disturbance set $\mathbb{W}$ are bounded polyhedra with non-empty interiors containing the origin. Next, let $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and let $\sigma \in \mathcal{K}$.

Definition 1. *A function $V : \mathbb{R}^n \to \mathbb{R}_+$ that satisfies*

$$\alpha_1(\|\hat{x}\|) \leq V(\hat{x}) \leq \alpha_2(\|\hat{x}\|), \quad \forall \hat{x} \in \mathbb{X} \subseteq \mathbb{R}^n \tag{11}$$

and for which there exists a possibly set-valued control law $\pi : \mathbb{R}^n \rightrightarrows \mathbb{U}$ such that

$$\begin{aligned} V(A_{di}\hat{x}_k + B_{di}u_k + w_k + L_i(y - \hat{y})) - V(\hat{x}) \leq -\alpha_3(\|\hat{x}\|) + \sigma(\|w\|), \\ \forall \hat{x} \in \mathbb{X}, \forall u \in \pi(\hat{x}), \forall w \in \mathbb{W}, \end{aligned} \tag{12}$$

is called an input-to-state stability control Lyapunov function (ISS-CLF) *in $\mathbb{X}$ for system* (6) *and disturbances in $\mathbb{W}$.*

ISS theory (see [8]) can be used to derive an input-to-state stabilizing predictive control scheme with improved disturbance rejection, as done in [7], where this property is refereed to as *optimized ISS*.

As such, let $\mathbb{W}$ be a convex hull of the vertices $w^e, e = 1, \ldots, E$, and let λ_k^e, $k \in \mathbb{Z}_+$, be optimization variables associated with each vertex w^e. Let $J(\lambda^1, \ldots, \lambda^E, \lambda) : \mathbb{R}_+^E \times \mathbb{R}_+ \to \mathbb{R}_+$ be a strictly convex, radially unbounded function (i.e. $J(\cdot)$ tends to infinity when its arguments tend to infinity) and let $J(\lambda^1, \ldots, \lambda^E, \lambda) \to 0 \Rightarrow \lambda^e \to 0$ for all $e = 1, \ldots, E$ and $\lambda \to 0$, and $J(0, \ldots, 0, 0) = 0$.

Choose off-line a CLF $V(\cdot)$ for system (6) without disturbances and let $\alpha_3 \in \mathcal{K}_\infty$ and $\hat{x} \in \mathbb{X}$ be given. At each control sampling instant $k \in \mathbb{Z}_+$ the one step ahead ISS MPC controller solves the following problem.

Problem 1. At time $k \in \mathbb{Z}_+$ obtain the observed state $\hat{x}_k$ and minimize the cost function $J(\lambda_k^1, \ldots, \lambda_k^E, \lambda_k)$ over $u_k, \lambda_k^1, \ldots, \lambda_k^E$ and λ_k, subject to the constraints

$$u_k \in \mathbb{U},\ (A_{di}\hat{x}_k + B_{di}u_k + L_i(y - \hat{y})) \in \mathbb{X},\ \lambda_k^e \geq 0, \lambda_k \geq 0, \tag{13a}$$

$$V(A_{di}\hat{x}_k + B_{di}u_k + L_i(y - \hat{y})) - V(\hat{x}_k) + \alpha_3(\|\hat{x}_k\|) \leq \lambda_k, \tag{13b}$$

$$V(A_{di}\hat{x}_k + B_{di}u_k + L_i(y - \hat{y}) + w^e) - V(\hat{x}_k) + +\alpha_3(\|\hat{x}_k\|) \leq \lambda_k^e, \tag{13c}$$

for all $e = 1, \ldots, E$. □

Let $\pi(\hat{x}_k) := \{u_k \in \mathbb{R}^m \mid \exists \lambda_k, \lambda_k^e, e \in \mathbb{Z}_{[1,E]}$ s.t. (13) holds$\}$ and let the definition $\phi_{\text{cl}}(\hat{x}_k, \pi(\hat{x}_k), w_k) := \{A_{di}\hat{x}_k + B_{di}u_k + w_k + L_i(y - \hat{y}) \mid u_k \in \pi(\hat{x}_k)\}$ denote the difference inclusion corresponding to system (6) in closed-loop with the set of feasible solutions obtained by solving Problem 1 at each sampling instant $k \in \mathbb{Z}_+$.

Next, the main robust stability result in terms of ISS is stated. This result is an adaptation of the main result in [7], to fit the relaxation (13b) of Problem 1, i.e., $\lambda_k = 0$ for all $k \in \mathbb{Z}_+$ corresponds to the problem considered in [7].

Theorem 2. *Let $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, a continuous and convex CLF $V(\cdot)$ and a cost $J(\cdot)$ be given. Suppose that Problem 1 is feasible for all states x in $\mathbb{X}$ and assume that $\lim_{k\to\infty} \lambda_k^* = 0$. Then, the trajectories generated by the difference inclusion*

$$\hat{x}_{k+1} \in \phi_{cl}(\hat{x}_k, \pi(\hat{x}_k), w_k), \quad k \in \mathbb{Z}_+, \tag{14}$$

with initial state $\hat{x}_0 = x_0 \in \mathbb{X}$ converge in finite time to a robustly positively invariant subset of $\mathbb{X}$, in which the difference inclusion is ISS for disturbances in $\mathbb{W}$.

The proof of Theorem 2 follows from standard arguments employed in proving ISS and Lyapunov stability and is therefore omitted here. The interested reader is referred to [7] and [9] for more details. Advantageous properties of the proposed robust controller are that ISS is guaranteed for any (feasible) solution of the optimization problem, state and input constraints can be explicitly accounted for, and feedback to disturbances is provided actively, on-line. The key of the stability proof is the limiting condition $\lim_{k\to\infty} \lambda_k^* = 0$ and a non-conservative solution for guaranteeing this condition is provided in [10].

The developed robust MPC scheme for the constrained system (6) can be implemented by solving a single LP during each control cycle using an infinity-norm based candidate CLF as shown in [5] and is omitted here.

4 Experimental results

This section presents the validation of the proposed networked one step ahead robust predictive control strategy based on the designed Luenberger observer investigated on the M220 Industrial plant emulator. The system has two working modes: the contact mode, when the torque is transmitted to the wheels, and the backlash (non-contact) mode, when no torque is transmitted from the engine to the driving wheels. For the contact mode the system matrices yield

$$A_{c1} = \begin{bmatrix} -12.08 & 10.13 & -5.04 \\ 0.16 & 2.47 & 311.81 \\ 0.25 & -1 & 0 \end{bmatrix}, B_c = \begin{bmatrix} 2383.8 \\ 0 \\ 0 \end{bmatrix}, C_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \quad (15)$$

In a similar way, the non-contact mode is characterized by transmitting no torque from the engine to the wheels, which yields the following system matrix

$$A_{c2} = \begin{bmatrix} -9.54 & 0 & 0 \\ 0 & -1.85 & 0 \\ 0.25 & -1 & 0 \end{bmatrix}, \quad (16)$$

and matrices B_c and C_c remain the same as for the contact mode. Please note that on a real automotive drivetrain only angular velocities can be measured, so the full state will be estimated by the designed observer.

The sampling period of the system was chosen as $T_s = 4$ms and the upper bound of the delays that are induced by CAN was calculated using the methodology described in [11], resulting that $\tau^{max} = 2T_s = 0.008$s for each channel. Then, the bounds of the disturbances (calculated using the methodology describe in Section 2) are explicitly taken into account by the robust one step ahead predictive control strategy described in Section 3. The delays are time-varying and uniformly distributed in the interval $[0, \tau^{max}]$.

The control objective is to reach a desired wheel angular velocity in a short time and to increase the passenger comfort by reducing the backlash effect. The axle wrap is calculated as the difference between the engine speed (divided by the total transmission ratio) and the wheel speed, and it is used as a measure of the driveline oscillations.

The experimental results in which the wheel angular velocity reference goes from 0 rad/s to 20 rad/s are presented in Fig. 3. The wheel angular velocity is illustrated in Fig. 3 top right, where it can be seen that the system reaches the reference in a short time, having almost no overshoot when it approaches the reference wheel angular velocity. When the system is in the non-contact mode there is no torque transmitted to the driving wheels because of the backlash, so the wheel angular velocity is equal to 0. After the system enters the contact mode (see Fig. 3 bottom right), the wheel angular velocity starts to increase.

The worst case time needed for computation of the control input for the proposed predictive controller was less than 0.7ms, which meets the timing constraints.

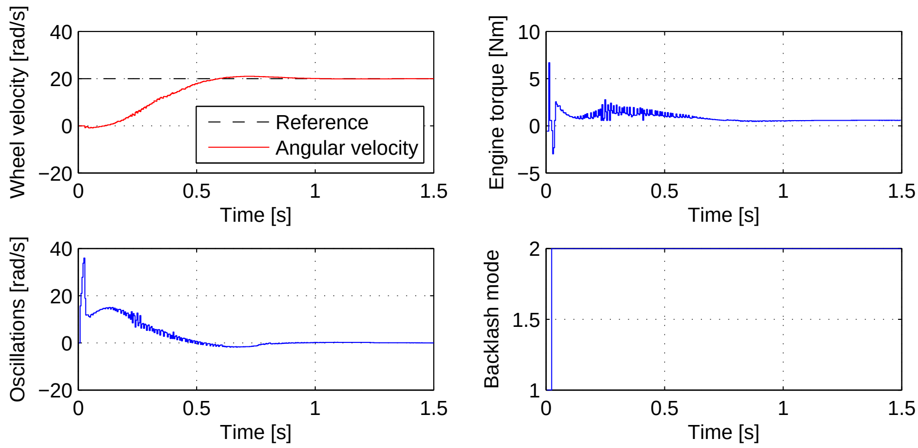

Fig. 3. a) Wheel angular velocity; b) Engine torque (control signal); c) Speed difference; d) Backlash mode.

The differences between the real states of the emulator and the estimated states are illustrated in Fig. 4 in which it can be seen that the estimated values of the states provided by the observer tend asymptotically to the real states in a short amount of time.

5 Conclusions

The designed robust one step ahead MPC strategy can make use of a Luenberger observer to obtain the full state while handling the performance/physical constraints and explicitly taking into account the disturbances caused by the time-varying delays. Also, a flexible control Lyapunov function was employed to obtain a non-conservative ISS stability guarantee for the developed one step ahead MPC scheme. The proposed control strategy was tested on a real-time simulation test-bench including CAN communications and the results obtained illustrate that the proposed controller has good performances and it meets the required timing constraints.

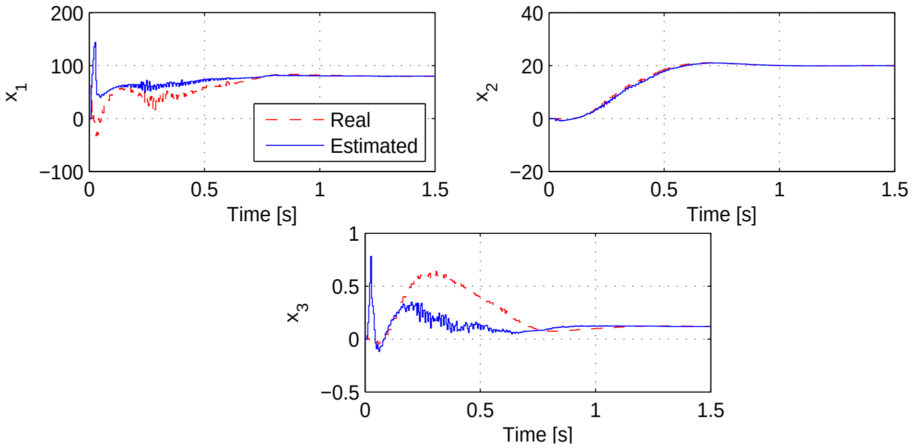

Fig. 4. Evolution of the states.

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